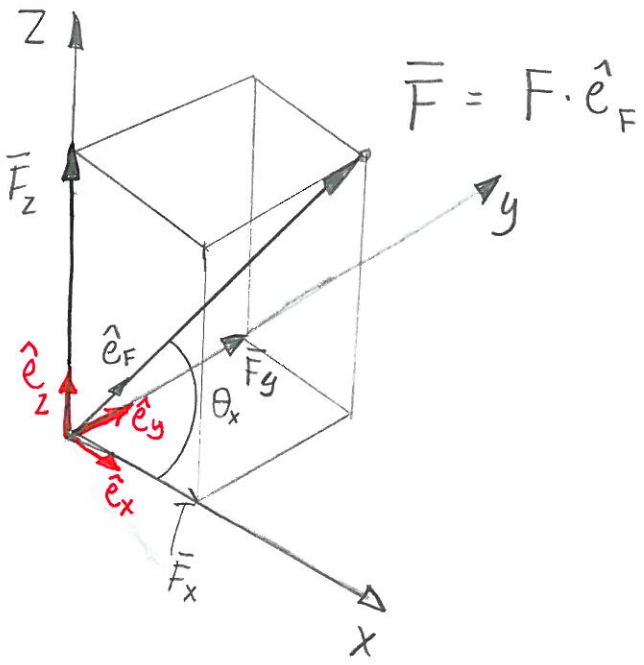
Krafter i 3 dimensioner



Höger koordinat-system

$$\bar{F} = (F_x, F_y, F_z) = F_x \hat{e}_x + F_y \cdot \hat{e}_y + F_z \hat{e}_z$$

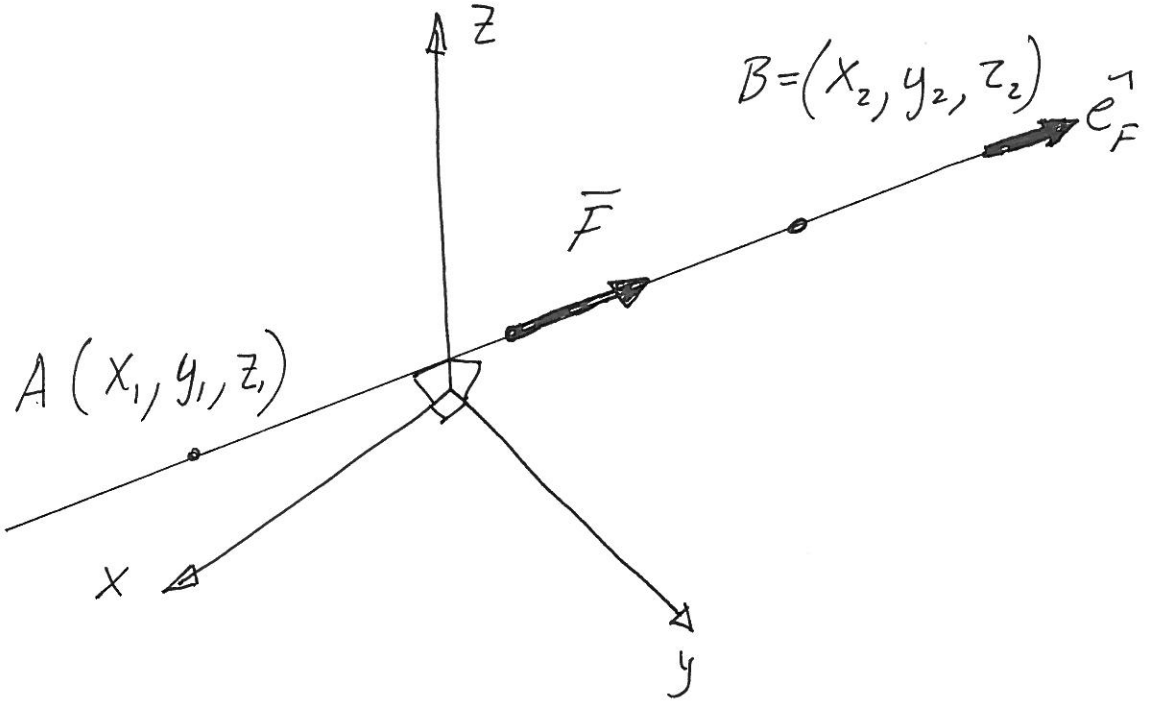
$$\bar{F} = (\bar{F}_x + \bar{F}_y + \bar{F}_z$$

$\bar{F}_x, \bar{F}_y, \bar{F}_z$ = komposanter

F_x, F_y, F_z = komponenter

$$F = |\bar{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$\bar{F} = F \cos\theta_x \hat{e}_x + F \cos\theta_y \hat{e}_y + F \cdot \cos\theta_z \cdot \hat{e}_z$$



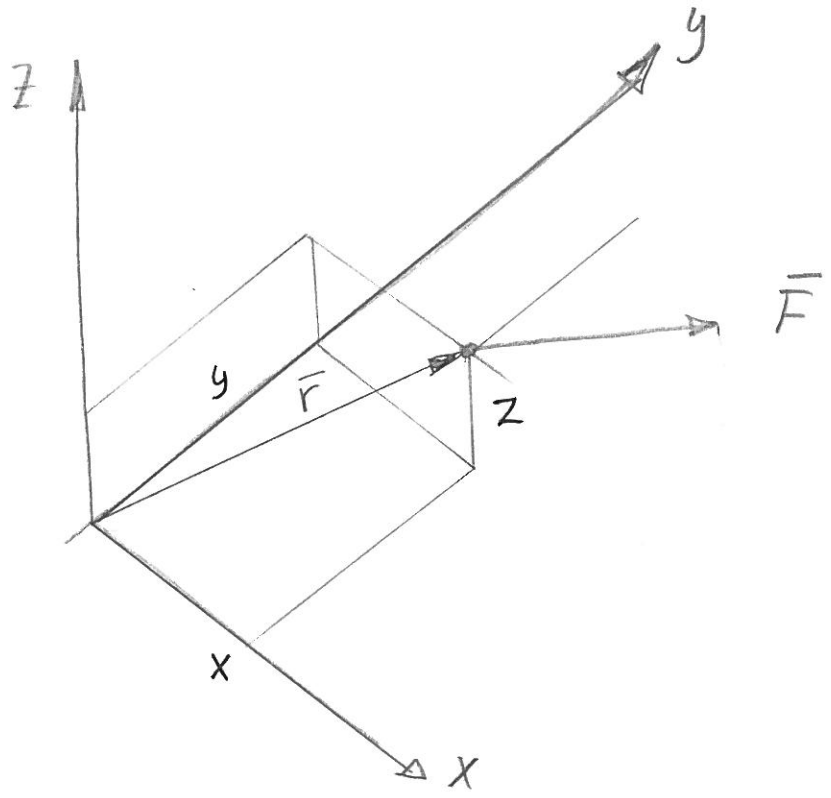
$$\bar{F} = F \cdot \hat{e}_F$$

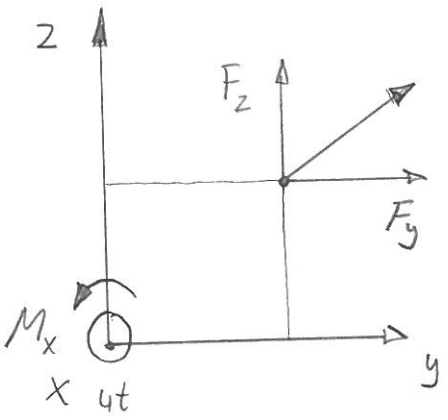
$$\overline{AB} = (x_2 - x_1)\hat{e}_x + (y_2 - y_1)\hat{e}_y + (z_2 - z_1)\hat{e}_z$$

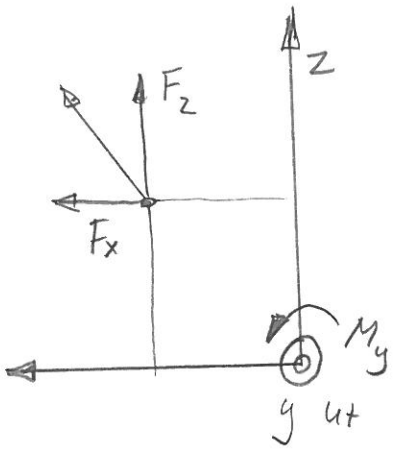
$$= \left((x_2 - x_1) + (y_2 - y_1), (z_2 - z_1) \right)$$

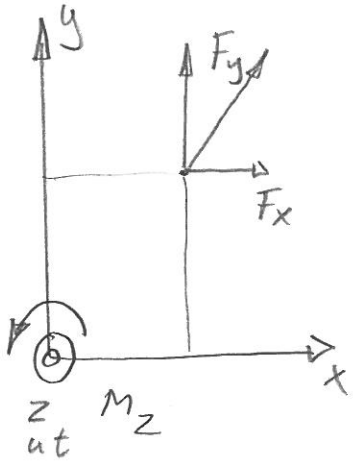
$$|\overline{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\hat{e}_F = \frac{\overline{AB}}{|AB|} = \frac{\left((x_2 - x_1), (y_2 - y_1), (z_2 - z_1) \right)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$









$$M_x = F_z \cdot y - F_y \cdot z = y \cdot F_z - z \cdot F_y$$

$$M_y = F_x \cdot z - F_z \cdot x = z \cdot F_x - x F_z$$

$$M_z = F_y \cdot x - F_x \cdot y = x \cdot F_y - y \cdot F_x$$

$$\overline{M} = \left(M_x, M_y, M_z \right)$$

Vektorprodukt Kryssprodukt

RÄKNEREGEL

$$\bar{A} \times \bar{B} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \begin{matrix} \hat{e}_x & \hat{e}_y \\ A_x & A_y \\ B_x & B_y \end{matrix} = \; - \;$$

$$= \left(A_y \cdot B_z - B_y \cdot A_z\right)\hat{e}_x + \left(A_z \cdot B_x - B_z \cdot A_x\right)\hat{e}_y +$$

$$+ \left(A_x B_y - B_x A_y\right)\hat{e}_z = -\bar{B} \times \bar{A}$$

Moment $\bar{M} = \bar{r} \times \bar{F}$

$\bar{r}$: lägevektor, $\bar{F}$: kraftvektor

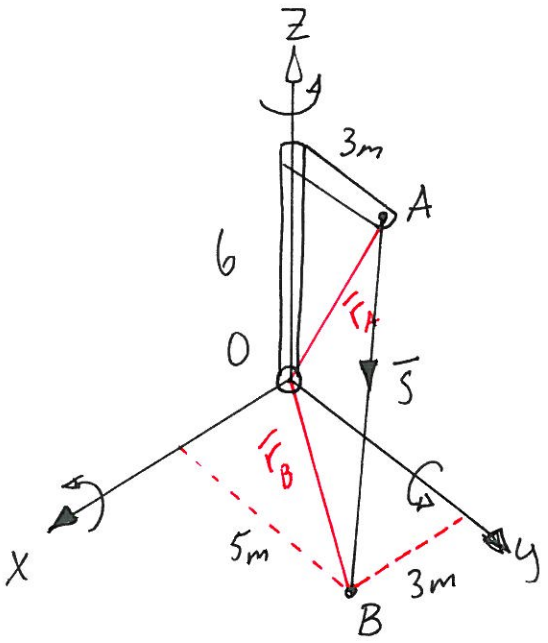
$$\bar{r} = (x, y, z)$$

$$\bar{F} = (F_x, F_y, F_z)$$

$$\bar{M} = \bar{r} \times \bar{F} = \begin{vmatrix} e_x & e_y & e_z \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} = \begin{matrix} \left((y \cdot F_z - z F_y), (z \cdot F_x - x \cdot F_z), (x \cdot F_y - y \cdot F_x)\right) \\ \left((F_z \cdot y - F_y \cdot z), (F_x \cdot z - F_z \cdot x), (F_y \cdot x - F_x \cdot y)\right) \end{matrix}$$

$$(M_x, M_y, M_z)$$

Ex



Stag AB spännkraft 1,4kN.

a) Bestäm $\bar{S}$ på A som vektor

$$\bar{r}_{AB} = \bar{r}_B - \bar{r}_A = (3,5,0) - (0,3,6)$$
$$= (3,2,-6)\,m$$

Enhetsvektorn

$$\hat{e}_{AB} = \frac{\bar{r}_{AB}}{|r_{AB}|} = \frac{3,2,-6}{\sqrt{3^2+2^2+(-6)^2}} =$$

$$= \frac{3,2,-6}{\sqrt{9+4+36}} = \frac{(3,2,-6)}{7}$$

$$\bar{S} = S \cdot \hat{e}_{AB} = \frac{S(3,2,-6)}{7} = \frac{1400}{7}(3,2,-6)$$

$$\bar{S} = 200(3,2,-6)\,N = (600, 400, -1200)\,N$$

b) Bestäm momentet map origo

$$\bar{M}_o = \bar{r}_A \times \bar{S} = 200 \cdot \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_y \\ 0 & 3 & 6 \\ 3 & 2 & -6 \end{vmatrix} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ 0 & 3 & 6 \\ 600 & 400 & -1200 \end{vmatrix}$$

$$= 200(3\cdot-6 - 6\cdot2,\ 6\cdot3 - 0\cdot-6,\ 0\cdot2 - 3\cdot3)$$
$$= 200(-30, 18, -9)\,Nm = (-6000, 3600, -1800)\,Nm$$